THE DYNAMICS OF THE QUADRATIC FAMILY

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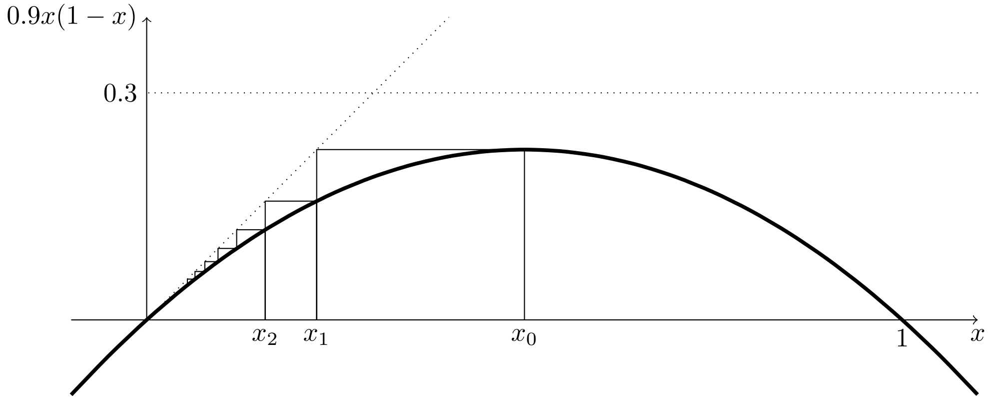
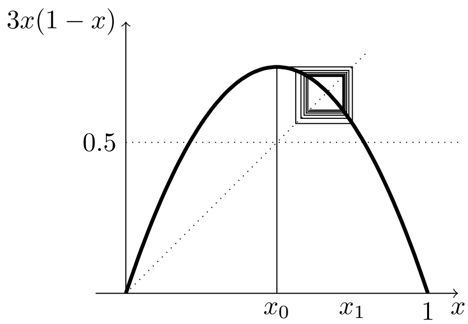
Let $\mu > 0$ and $f_\mu : \mathbb{R} \to \mathbb{R}$ be the map $x \to \mu x(1-x)$. For a given number x_0 we consider the sequence which is recursively defined by

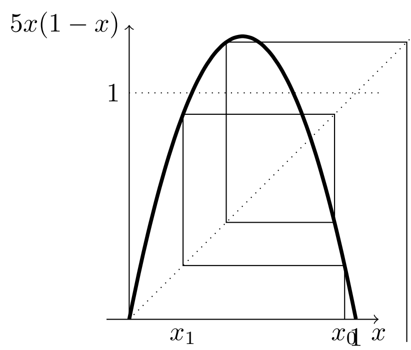
$$x_{n+1} = \mu x_n(1-x_n).$$

We visualize the dynamics for the value $\mu = 0.9$ in the following picture: Starting from a point on the x axis we draw a vertical line to the graph of the function, then a horizontal line to the diagonal, and then a vertical line to the graph and so on.



Here the points converge to 0. For $\mu = 3$ we see convergence to the intersection of the diagonal and the graph in the picture, which corresponds to convergence to the fixed point $x = \frac{2}{3}$ (which is mapped to itself). For $\mu = 5$ we see a sequence tending to $-\infty$.





We will see that we can get a good understanding of the behavior of the sequences. There are fixed points, period sequences and chaotic dynamics. The main tool will be the intermediate value theorem.