Divisibility Rules

1) Construct a 10-digit number divisible by 9 using only the digits 0 and 5.

   Source: Projekt MmF

2) What least number of times should the number 2013 be written to get a number which is divisible by 9? (Examples: 2013, 20132013, …)

   Source: [3], #64504

3) The value of the expression 1 + 2 + 3 + 5 + 6 + 7 + 8 + 9 is 45. Is it possible to change the value to 18 by replacing some of the + signs by - signs?

   Source: [2], №67

4) Show that neither 364782634 nor 2626840 is a square number.

   Source: Projekt MmF

5) Can a number whose decimal notation is used once - 1, twice - 2, three times - 3 and four times - 4 be the square of the number?

   Source: [4], 11.20

6) Prove that the number
   a) $21^{20} + 29^{21}$ is divisible by 10
   b) $2^{101} + 3^{1001}$ is divisible by 5
   c) $6^{2022} - (4^{2021} + 2^{2021})$ is divisible by 10.

   Source: Projekt MmF

7) Vasya wrote an example on the board for multiplying two two-digit numbers, and then replaced all the digits with letters, using the same letter for the same digit, and different letters for different digits. The result is AB × VG = DDEE.
   Prove that Vasya made a mistake somewhere.

   Source: [3], #30366

8) Find all numbers of the form $\overline{13xy45z}$ that are divisible by 792.

   Source: [3], #60793

9) Alice and Bob are writing a 19-digit number using only the digits 1, 2 and 4. Alice writes the first digit, Bob the second, Alice the third, and so on. Alice's goal is to get a number that is divisible by 3. Can Bob prevent it?

Source: [2], №95

10) We know that the number 35! Is equal to 10333147966386144929*66651337523200000000

Find the missing digit *.

Source: [3], #35012

11) For the number $a_0 = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 2022$ we count the sum of its digits and obtain the number a_1. For the number a_1 we also count the sum of its digits and receive a number a_2. We repeat this process until we get a 1-digit number. Which digit do we get in the end?

Hint: Simplify the problem. Try to find the final digit for the numbers $1 \cdot 2 \cdot \ldots \cdot 6$ and $1 \cdot 2 \cdot \ldots \cdot 10$.

Source: [2], №94

Bibliography

This problem set has been composed by the team of Projekt MmF. The sources of all problems are given. The translations and slight adaptations are due to us.

[1] Kangaroo (2015 - 2021), in ukrainian:
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